Score: ______ out of 10.

Math 201 - Quiz #6 Name: key

1. **Solve as few or as many as you think you need to maximize your score. Please put an X through the parts you do not want graded.**

   (a) Find interval(s) where f is **increasing**, interval(s) where f is **decreasing**, and find any local maximum and local minimum **value(s)** of f if:

$$f(x) = \frac{x^2}{x^2+3}.$$

$$f'(x) = \frac{(x^2+3)2x - x^2(2x)}{(x^2+3)^2} = \frac{\cancel{2x^3} + 6x - \cancel{2x^3}}{(x^2+3)^2} = \frac{6x}{(x^2+3)^2}$$

Critical numbers: $f'(x) = 0$, $6x = 0$, $x = 0$ | No vals for $f'(x)$ does not exist $\times$

f': $f'(-1) = \frac{-}{+} = -$; $f(1) = \frac{+}{+} = +$

0

intervals: $(-\infty, 0)$; $(0, \infty)$

f is: decreasing ; increasing

local min at $(0, f(0)) = \boxed{(0,0)}$

   (b) Find interval(s) where f is **concave up**, interval(s) where f is **concave down**, and find any inflection **points**:

$$f(x) = \frac{x^2}{x^2+3}.$$

$$f''(x) = \frac{(x^2+3)^2 6 - 6x\left(2(x^2+3)'(2x)\right)}{(x^2+3)^4}$$

$$= \frac{6(x^2+3)\left[(x^2+3) - 2x(2x)\right]}{(x^2+3)^4}$$

$$= \frac{6\cancel{(x^2+3)}\left[-3x^2+3\right]}{(x^2+3)^{\cancel{4}\,3}} = \frac{18(1-x^2)}{(x^2+3)^3}$$

$$f''(x) = 0 = \frac{18(1-x^2)}{(x^2+3)^3} \longrightarrow \begin{array}{l} 1-x^2=0 \\ 1 = x^2 \\ x = \pm\sqrt{1} = \pm 1 \end{array}$$

f'': $f''(-2) = \frac{-}{+}$ $-$; $f''(0) = \frac{+}{+}$ $+$; $f(2) = \frac{-}{+}$ $-$

-1 ; 1

intervals: $(-\infty, -1)$; $(-1, 1)$; $(1, \infty)$

f is: concave down ; concave up ; concave down

inflection pts at:

$x = 1 \rightarrow \boxed{(1, 1/4)}$

$x = -1 \rightarrow \boxed{(-1, 1/4)}$

(c) $\lim_{x\to 0} \frac{\sin(3x)+\sin(4x)}{\tan(5x)}$ ← type $\frac{0}{0}$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{\frac{d}{dx}\left(\sin(3x)+\sin(4x)\right)}{\frac{d}{dx}\left(\tan(5x)\right)}$$

$$= \lim_{x\to 0} \frac{3\cos(3x)+4\cos(4x)}{5\sec^2(5x)} = \frac{3\cdot 1+4\cdot 1}{5\cdot\left(\frac{1}{1^2}\right)} = \frac{7}{5}$$

answer: $7/5$

(d) $\lim_{x\to\infty} \frac{(\ln(x))^2}{x}$ ← type $\frac{\infty}{\infty}$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{2(\ln x)^1\left(\frac{1}{x}\right)}{1}$$

$$= \lim_{x\to\infty} \frac{2\ln x}{x}$$ ← type $\frac{\infty}{\infty}$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{2\left(\frac{1}{x}\right)}{1} = \lim_{x\to\infty} \frac{2}{x} = 0$$

answer: 0

(e) $\lim_{x\to\infty} x^{1/x}$ ← type ∞^0

$$= \lim_{x\to\infty} e^{\ln(x^{1/x})}$$

$$= \lim_{x\to\infty} e^{\left(\frac{1}{x}\right)\ln(x)} = e^{\lim_{x\to\infty} \frac{\ln x}{x}}$$ ← type $\frac{\infty}{\infty}$

$$\overset{L'H}{=} e^{\lim_{x\to\infty} \frac{\left(\frac{1}{x}\right)}{1}}$$

$$= e^0 = 1$$

answer: 1