

key

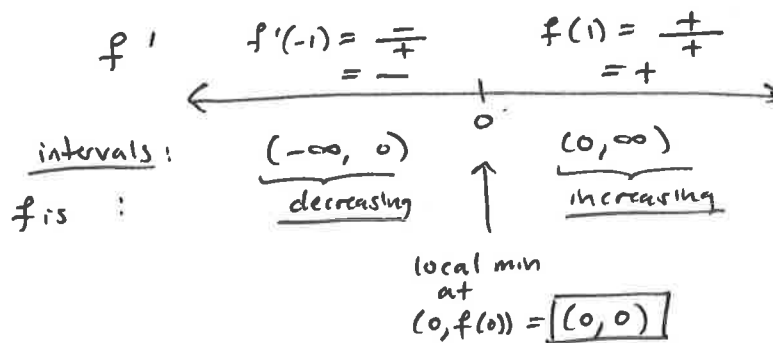
1. Solve as few or as many as you think you need to maximize your score. Please put an X through the parts you do not want graded.

- (a) Find interval(s) where f is **increasing**, interval(s) where f is **decreasing**, and find any local maximum and local minimum value(s) of f if:

$$f(x) = \frac{x^2}{x^2 + 3}$$

$$f'(x) = \frac{(x^2 + 3)2x - x^2(2x)}{(x^2 + 3)^2} = \frac{\cancel{2x^3} + 6x - \cancel{2x^3}}{(x^2 + 3)^2} = \frac{6x}{(x^2 + 3)^2}$$

Critical numbers: $f'(x) = 0$ | No value for $f'(x)$ does not exist
 $6x = 0$
 $x = 0$ X



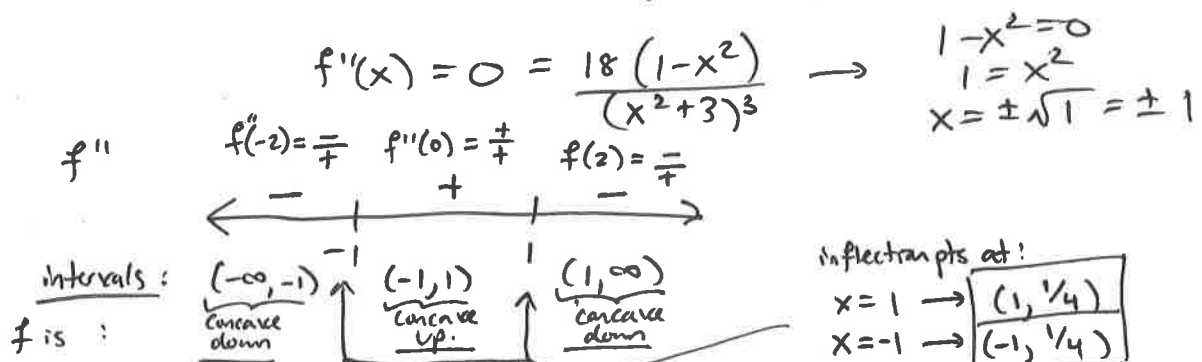
- (b) Find interval(s) where f is **concave up**, interval(s) where f is **concave down**, and find any inflection points:

$$f(x) = \frac{x^2}{x^2 + 3}$$

$$f''(x) = \frac{(x^2 + 3)^2 6 - 6x (2(x^2 + 3)'(2x))}{(x^2 + 3)^4}$$

$$= \frac{6(x^2 + 3)[(x^2 + 3) - 2x(2x)]}{(x^2 + 3)^4}$$

$$= \frac{6\cancel{(x^2 + 3)}[-3x^2 + 3]}{(x^2 + 3)^4 3} = \frac{18(1 - x^2)}{(x^2 + 3)^3}$$



$$(c) \lim_{x \rightarrow 0} \frac{\sin(3x) + \sin(4x)}{\tan(5x)} \leftarrow \text{type } \frac{0}{0}$$

$$\begin{aligned} L'H &= \lim_{x \rightarrow 0} \frac{\frac{d}{dx} (\sin(3x) + \sin(4x))}{\frac{d}{dx} (\tan(5x))} \\ &= \lim_{x \rightarrow 0} \frac{3\cos(3x) + 4\cos(4x)}{5\sec^2(5x)} = \frac{3 \cdot 1 + 4 \cdot 1}{5 \cdot (\frac{1}{1^2})} = \frac{7}{5} \end{aligned}$$

answer:

$$\frac{7}{5}$$

$$(d) \lim_{x \rightarrow \infty} \frac{(\ln(x))^2}{x} \leftarrow \text{type } \frac{\infty}{\infty}$$

$$\begin{aligned} L'H &= \lim_{x \rightarrow \infty} \frac{2(\ln x) \left(\frac{1}{x}\right)}{1} \\ &= \lim_{x \rightarrow \infty} \frac{2 \ln x}{x} \leftarrow \text{type } \frac{\infty}{\infty} \\ L'H &= \lim_{x \rightarrow \infty} \frac{2 \left(\frac{1}{x}\right)}{1} = \lim_{x \rightarrow \infty} \frac{2}{x} = 0 \end{aligned}$$

answer:

$$0$$

$$(e) \lim_{x \rightarrow \infty} x^{1/x} \leftarrow \text{type } \infty^0$$

$$\begin{aligned} &= \lim_{x \rightarrow \infty} e^{\ln(x^{1/x})} \\ &= \lim_{x \rightarrow \infty} e^{\left(\frac{1}{x}\right) \ln(x)} = e^{\lim_{x \rightarrow \infty} \frac{\ln x}{x}} \leftarrow \text{type } \frac{\infty}{\infty} \\ L'H &= e^{\lim_{x \rightarrow \infty} \frac{\left(\frac{1}{x}\right)}{1}} \\ &= e^0 = 1 \end{aligned}$$

answer:

$$1$$