Score:_______ out of 10.

Math 201 - Quiz #7 Name: Key

Solve as few or as many as you think you need to maximize your score. Please put an X through the problems you do not want graded.

1. Use the guidelines for curve sketching the produce a sketch (**draw this on the provided window**) for the function
$$f(x) = \frac{x}{x-1}.$$
To receive full credit on this problem you must cover all parts of the guidelines.

1. Domain: $\boxed{x \neq 1}$ OR: $\boxed{(-\infty, 1) \cup (1, \infty)}$

2. ~~x~~ Intercepts:

y-intercept: $(0, f(0)) = \boxed{(0,0)}$

x-intercept: $\frac{x}{x-1} = 0 \rightarrow x = 0$ so the point is the same as the y-intercept. $\boxed{(0,0)}$

3. Symmetry:
$f(-x) = \frac{-x}{-x-1}$ ⟵ NOT $f(x)$ OR $-f(x)$ so $\boxed{\text{no even/ODD symmetry.}}$

4. Asymptotes:

H.A: $\lim_{x\to\infty} \frac{x}{x-1} \overset{L'H}{=} \lim_{x\to\infty} \frac{1}{1} = 1 \longrightarrow \boxed{\text{H.A. @ } y = 1}$

$\lim_{x\to-\infty} \frac{x}{x-1} = 1$ ⟵ SAME

V.A.: The value $x=1$ makes the denominator of f 0, so it is potentially a vertical asymptote.

$\lim_{x\to 1^+} \frac{x}{x-1} = +\infty$ (x: +, $x-1$: +) AND $\lim_{x\to 1^-} \frac{x}{x-1} = -\infty$ (x: +, $x-1$: −)

5. Sign analysis on f':

$f'(x) = \frac{(x-1)(1) - x(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$

$f'(x) = 0$
$-1 = 0$ ⟵ NO SOLUTIONS

$f'(x)$ does not exist at $x=1$, but this is not in domain

f': − on $(-\infty, 1)$, − on $(1, \infty)$; open point at 1 ⟵ DOMAIN of f

f is decreasing on $(-\infty, 1)$, decreasing on $(1, \infty)$; ↑ asymptote here.

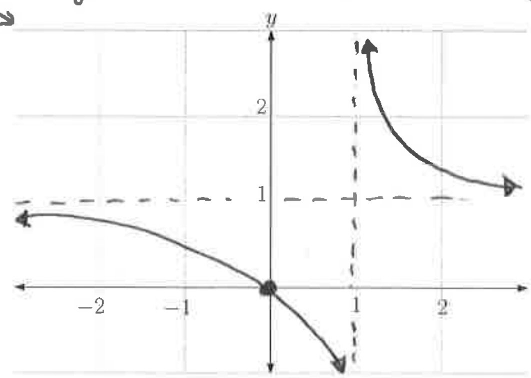
6. Sign analysis of f'': (use either Chain rule OR Quotient Rule!)

$f''(x) = \frac{0 - (-1)(2(x-1)'(1))}{(x-1)^4} = \frac{2(x-1)}{(x-1)^4} = \frac{2}{(x-1)^3}$

$f''(x) = 0$ if $2 = 0$ ⟵ NO SOLUTION!

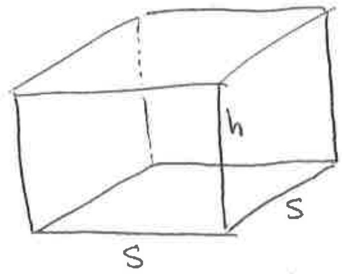
f'': $\frac{+}{(-)^3} = -$ on $(-\infty, 1)$; $\frac{+}{(+)^3}$ + on $(1, \infty)$

f is concave down on $(-\infty, 1)$, concave up on $(1, \infty)$.



2. A box with a square base and an open top must have a volume of 4 m^3. Find the dimensions of the box that minimize the amount of material used.

OPEN TOP!



Volume $= 4 = s^2h$

we want to MINIMIZE the material (or surface area)

Surface Area $= A = s^2 + 4sh$

bottom   4 sides.   N.B. NO TOP

Now since $4 = s^2h \rightarrow h = \frac{4}{s^2}$ so

$$A = s^2 + 4s\left(\frac{4}{s^2}\right) = s^2 + \frac{16}{s}$$

Now A is a function of just s! (one variable)

$$A(s) = s^2 + \frac{16}{s}$$

$$A'(s) = 2s - \frac{16}{s^2} = \frac{2s^3 - 16}{s^2}$$

$$A'(s) = 0 = \frac{2s^3 - 16}{s^2} \rightarrow \begin{aligned} 2s^3 - 16 &= 0 \\ 2s^3 &= 16 \\ s^3 &= 8 \\ s = \sqrt[3]{8} &= 2 \end{aligned}$$

A' $-$ 2 $+$

ABSOLUTE MIN (not just local min.)

if $s = 2 \rightarrow h = 4/s^2 = 4/2^2 = 4/4 = 1$

Dimensions: $\boxed{2\text{ m.} \times 2\text{ m.} \times 1\text{ m.}}$