Math 324
Additional Practice Problems for Exam 2

1. Suppose $\mathbf{y} \in \text{span}(\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\})$. Prove the collection $\{\mathbf{y}, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ is linearly dependent.
2. Prove or disprove: The collection $W = \{(x_1, 0, 0, x_4) : x_1, x_4 \in \mathbb{R}\}$ a subspace of $\mathbb{R}^4$
3. Prove or disprove: The collection $W = \{a_0 + a_3x^3 : a_0, a_3 \in \mathbb{R}\}$ is a subspace of P_3.
4. Prove or disprove: The collection $W = \{A \in M_{33} : \text{tr}(A) = 0\}$ is a subspace of M_{33}.
5. Prove or disprove: The collection $W = \{A \in M_{33} : \text{tr}(A) = 4\}$ is a subspace of M_{33}.
6. Prove or disprove: The collection of functions $W = \{f(x) = A\sin(x) + B\cos(x) : A, B \in \mathbb{R}\}$ is a subspace of $F(-\infty, \infty)$.
7. Let U and W be subspaces of some vector space V. Consider the sets:

$$\begin{aligned} U \cap W &= \{x : x \in U \text{ and } x \in W\} \\ U \cup W &= \{x : x \in U \text{ or } x \in W\} \\ U + W &= \{x + y : x \in U \text{ and } y \in W\} \end{aligned}$$

   Which of the above (if any) are subsets of V.

8. Let V be a vector space. Suppose $S, T \subseteq V$ (not necessarily subspaces). Prove each of the following:
   (a) If $S \subseteq T$, then $\text{span}(S) \subseteq \text{span}(T)$.
   (b) $\text{span}(\text{span}(S)) = \text{span}(S)$.
9. Suppose $T : V \to W$ is a linear transformation. Prove: If $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ is a collection of linearly independent vectors in V, then the collection $\{T(\mathbf{x}_1), T(\mathbf{x}_2), \ldots, T(\mathbf{x}_n)\}$ is linearly independent. Is the converse true? What can be said about dependence?