## Math 324

Exam 2 will cover section 4.1-4.5,4.7,4.8 and 8.1. Some fundamental topics are listed below:

- 1. Closure under vector addition, Closure under scalar multiplication.
- 2. Vector Spaces ( $\mathbb{R}^n$ ,  $P_n$ ,  $M_{mn}$ ,  $\mathbb{R}^\infty$ ,  $P_\infty = \mathbb{R}[x]$ ,  $F(-\infty, \infty)$ , etc.)
- 3. Subspaces (how to show W is a subspace of V)
- 4. Geometry of subspaces in  $\mathbb{R}^n$ .
- 5. Linear combinations
- 6. Span
- 7. Linear Independence/Dependence
- 8. Basis (you should know the standard basis of  $\mathbb{R}^n$ ,  $P_n$ ,  $M_{mn}$ , but also how to show something is a basis, especially in  $\mathbb{R}^n$ .)
- 9. Dimension
- 10. Finite Dimensional vs. Infinite Dimensional Vector Spaces
- 11. Coordinate vectors
- 12. The fundamental subspaces associate to a matrix A: Null(A), Row(A), Col(A),  $Null(A^T)$ :
  - (a) What are they
  - (b) How to find a basis for each of these subspaces
  - (c) Compute the dimension of each. What is rank(A)? What is nullity(A)?
  - (d) Rank-Nullity Theorem
  - (e) Orthogonal Complements
  - (f) Basis for the Span(S) if  $S = {\mathbf{x}_1, \dots, \mathbf{x}_n}$ .
  - (g) Good idea to see the extended list of Equivalent Statements so far (see page 245), but you do not need to have this memorized.
- 13. Linear Transformations
  - (a) Show  $T: V \to W$  is a linear transformation
  - (b) If  $T : \mathbb{R}^n \to \mathbb{R}^m$  is a linear transformation, find the associated matrix [T].
  - (c) Find  $\ker(T)$
  - (d) Find range(T) (the image im(T))
  - (e) What is rank(T)? What is nullity(T)?
  - (f) Rank-Nullity Theorem