NOTE: When scaling we need to take reciprocals. See why in the following example.

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -6 & -3 & 9 \\ 3 & -12 & 3 & 33 \\ 0 & -4 & -2 & 5 \end{vmatrix} \xrightarrow{R2 \leftrightarrow R3}$$

will introduce a −

$$- \begin{vmatrix} 1 & 2 & 1 & 0 \\ 3 & -12 & 3 & 33 \\ 0 & -6 & -3 & 9 \\ 0 & -4 & -2 & 5 \end{vmatrix} \xrightarrow{R2 \to \frac{1}{3} R2}$$

NOTE: I am scaling by $\frac{1}{3}$ so I need to multiply the result by 3. {why??}

Recall: $\det(B) = k \det(A)$

new matrix. old matrix

So when writing out steps: $\det(A) = \frac{1}{k} \det(B)$

old matrix (above) new matrix (below)

$$(-)(3) \begin{vmatrix} 1 & 2 & 1 & 0 \\ 1 & -4 & 1 & 11 \\ 0 & -6 & -3 & 9 \\ 0 & -4 & -2 & 5 \end{vmatrix} \xrightarrow{R2 \to R2 - R1}$$

no change.

$$-3 \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -6 & 0 & 11 \\ 0 & -6 & -3 & 9 \\ 0 & -4 & -2 & 5 \end{vmatrix} \xrightarrow{R3 \to \frac{1}{3} R3}$$

NOTE: I am scaling by $1/3$ so I need to multiply the new result by 3

$$(3)(-3) \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -6 & 0 & 11 \\ 0 & -2 & -1 & 3 \\ 0 & -4 & -2 & 5 \end{vmatrix} \xrightarrow{R4 \to R4 - 2R3}$$

$$(3)(-3) \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -6 & 0 & 11 \\ 0 & -2 & -1 & 3 \\ 0 & 0 & 0 & -1 \end{vmatrix} \xrightarrow{R3 \to R3 - \frac{1}{3} R2}$$

$$(3)(-3) \begin{vmatrix} 1 & 2 & 1 & 0 \\ 0 & -6 & 0 & 11 \\ 0 & 0 & -1 & -2/3 \\ 0 & 0 & 0 & -1 \end{vmatrix} = (3)(-3)(-6)(-1)(-1) = \boxed{54}$$