Math 324 - Additional Problems HW#9

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} y \\ x \end{bmatrix}$$

   (a) Is T a linear transformation?

   (b) Describe geometrically what T does to points in $\mathbb{R}^2$.

   (c) If T is a linear transformation, find the associated standard matrix $[T]$.

2. Suppose θ is some fixed real number. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1\cos(\theta) - x_2\sin(\theta) \\ x_1\sin(\theta) - x_2\cos(\theta) \end{bmatrix}$$

   (a) Is T a linear transformation?

   (b) If T is a linear transformation, find the associated standard matrix $[T]$.

   (c) Can you see geometrically what T does to points in $\mathbb{R}^2$?

3. Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ 0 \\ x_3 \end{bmatrix}$$

   (a) Is $\begin{bmatrix} 0 \\ \pi \\ 0 \end{bmatrix} \in \ker(T)$? Explain your answer.

   (b) Is $\begin{bmatrix} \pi \\ \pi \\ \pi \end{bmatrix} \in \text{range}(T)$? Explain your answer.

   (c) Find $\ker(T)$

   (d) Find $\text{range}(T)$

4. Consider the linear transformation $T : M_{22} \to \mathbb{R}$ given by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \text{tr}(A) = a + d.$$

   (a) Is $\begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix} \in \ker(T)$? Explain your answer.

   (b) Is $5 \in \text{range}(T)$? Explain your answer.

   (c) Find $\ker(T)$

(d) Find range(T)

5. Consider the linear transformation $D : P_2 \to P_1$ given by

$$D(\mathbf{p}) = p'(x).$$

(a) Find ker(D)

(b) Find range(D)