EXAM 1

Score: __________ out of 100

Math 324 - Linear Algebra **Name:** __________________________________

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please $\boxed{\text{box/circle}}$ or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 8 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Circle your answer for each of the following:

   (a) | True | False | A homogeneous linear system is always be consistent.

   (b) | True | False | A linear system may have exactly 5 solutions.

   (c) | True | False | If A is a 4×7 matrix and B is a 7×7 matrix, then BA exsits.

   (d) | True | False | If B has a column of zeros, then so does AB (if this product is defined).

   (e) | True | False | If A is an $n \times n$ matrix, then $\mathrm{tr}(5A) = 5\mathrm{tr}(A)$.

   (f) | True | False | If A is invertible, then so is A^{T}.

   (g) | True | False | The sum of two invertible matrix of the same size must be invertible.

   (h) | True | False | Every elementary matrix is invertible.

   (i) | True | False | If A is an $n \times n$ matrix, then $\det(A^2) = (\det(A))^2$.

   (j) If A is a 3×3 invertible matrix then $\det(3A) =$ | $3\det(A)$ | $9\det(A)$ | $27\det(A)$ |

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2. (a) Show that $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 3 & -4 & 1 \\ -1 & -5 & -1 & 1 \end{bmatrix}$ has reduced row echelon (RREF) form $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

   You must show each step of your work for full credit.

   (b) Use part (a) to find the solution of the system:

$$\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & + & 3x_3 & + & 2x_4 & = & 0 \\ x_1 & + & 3x_2 & - & 4x_3 & + & x_4 & = & 0 \\ -x_1 & - & 5x_2 & - & x_3 & + & x_4 & = & 0 \end{array}$$

3. Let

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}$$

Compute each of the following, or explain/show why it is NOT possible to compute.

(a) A^{T}

(b) $A + 3B$

(c) AB

(d) BA

(e) $\mathrm{tr}(A)$

(f) $\det(A)$

(g) A^{-1}

(h) $\det(B)$

(i) B^{-1}

4. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$

(a) Compute A^{-1}.

(b) Use the A^{-1} computed in part (a) to solve the equation

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

.

5. Compute the following determinant: $\begin{vmatrix} 2 & 0 & 2 & -1 \\ 2 & 0 & 6 & -1 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{vmatrix}$

6. Let $A = \begin{bmatrix} 3 & 1 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & -5 & 1 \end{bmatrix}$

(a) $\det(A) = \boxed{}$

(b) $\det(B) = \boxed{}$

For the remaining parts you may not calculate the matrices involved, and should use your answers from parts (a) and (b):

(c) $\det(A^{-1}) = \boxed{}$

(d) $\det(A^{\mathrm{T}}) = \boxed{}$

(e) $\det(A^2) = \boxed{}$

(f) $\det(B^{-1}A^{-1}A^{\mathrm{T}}A^2B) =$

answer: $\boxed{}$

For the following two problems 0 will be used to denote the zero matrix.

7. Show that if $A^5 - 2A^2 + A - I = \mathbf{0}$, then A^{-1} exists and $A^{-1} = A^4 - 2A + I$

8. Show that if $A^2 = \mathbf{0}$, then A is not invertible.