EXAM 2

Score: ________ out of 100

Math 324 - Linear Algebra **Name:** ______________________________

Read all of the following information before starting the exam:

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please $\boxed{\text{box/circle}}$ or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 7 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Circle your answer for each of the following:

   (a) True | False Every vector space is a subspace of itself.

   (b) True | False If V is a vector space and $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ is a collection of vectors in V, then Span(S) is always a subspace of V.

   (c) True | False The solution set of a consistent linear system $A\mathbf{x} = \mathbf{0}$ of m equations in n variables is a subspace of $\mathbb{R}^n$.

   (d) True | False The solution set of a consistent linear system $A\mathbf{x} = \mathbf{b}$ of m equations in n variables is a subspace of $\mathbb{R}^n$.

   (e) True | False If V is a vector space and $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ is a collection of linearly independent vectors in V, then for any nonzero scalar k, $\{k\mathbf{v}_1, k\mathbf{v}_2, \ldots, k\mathbf{v}_k\}$ is also a collection of linearly independent vectors in V.

   (f) True | False A set containing a single vector is always linearly independent.

   (g) True | False Every dependent set contains the zero vector.

   (h) True | False $\text{Row}(A)^\perp = \text{Null}(A)$.

   (i) True | False $\text{Span}(\{1, x, 1 - x\}) = P_2$.

   (j) True | False Let $T: V \to W$ be a linear transformation, then $\text{range}(T) = \text{im}(T) = \{T(\mathbf{x}) : \mathbf{x} \in V\}$.

2. Suppose a 5×9 matrix A has rank 4. Then

   (a) The dimension of the column space of A is

   (b) The dimension of the row space of A is

   (c) The dimension of the null space of A is

   (d) The dimension of the null space of A^T is

3. Show that $W = \{a_1 x + a_4 x^4 : a_1, a_4 \in \mathbb{R}\}$ forms a subspace of P_4.

4. Determine whether or not $S = \left\{ \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} \right\}$ a basis of $\mathbb{R}^3$.

5. Let $T : M_{22} \to \mathbb{R}^3$ defined by

$$T\left(\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

(a) Show that T is a linear transformation

(b) Find $\ker(T)$

6. Consider the matrix $A = \begin{bmatrix} 1 & -1 & 0 & -1 \\ -1 & 1 & 1 & 1 \end{bmatrix}$

(a) Find a basis for Null(A).

(b) Find a basis for Row(A)

(c) Find a basis for Col(A)

(d) Find a basis for Null(A^T)

(e) rank(A) = $\boxed{}$

(f) nullity(A) = $\boxed{}$

(g) nullity(A^T) = $\boxed{}$

(h) The columns of A span a $\boxed{}$ dimensional subspace of $\boxed{}$

(i) The rows of A span a $\boxed{}$ dimensional subspace of $\boxed{}$

7. **Solve 2 of the following problems**. Please put an X through the problem that you do not want graded (otherwise I will grade the first two problems worked on).

(a) Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$ be a collection of vectors in some vector space V. Suppose $\mathbf{v}_1 + 2\mathbf{v}_2 = \mathbf{v}_3 - \mathbf{v}_4$. Show that S is a set of linearly dependent vectors.

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation, and suppose $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$.

Let $\mathbf{x} \in \ker(T)$.

Compute $T\left(2\mathbf{x} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) =$

(c) Prove that for any matrix A, $\operatorname{rank}(A^T) = \operatorname{rank}(A)$.