# EXAM 3

**Score:** ________ out of 100

Math 324 - Linear Algebra **Name:** ______________________________

**Read all of the following information before starting the exam:**

- You have 50 minutes to complete the exam.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please $\boxed{\text{box/circle}}$ or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 7 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. Circle your answer for each of the following:

(a) True | False $P_n$ is isomorphic to $\mathbb{R}^n$.

(b) True | False $M_{mn}$ is isomorphic to $\mathbb{R}^{mn}$.

(c) True | False There is a subspace of $P_8$ isomorphic to $M_{22}$.

(d) True | False If $V$ is a finite dimensional vector space and $T : V \to V$ is an isomorphism, then $\ker(T) = \{\mathbf{0}\}$.

(e) True | False Every linear transformation $T : M_{22} \to \mathbb{R}^4$ is an isomorphism.

(f) True | False If $V$ and $W$ are finite dimensional and isomorphic vector spaces, then $\dim(V) = \dim(W)$.

(g) True | False $\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$ are similar matrices.

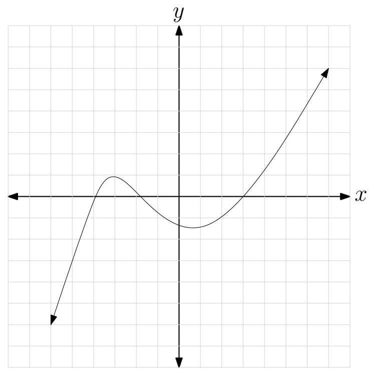
(h) True | False If $A$ is a $3 \times 3$ matrix with three eigenvalues $\lambda_1 = 1$, $\lambda_2 = 5$ and $\lambda_3 = 15$, then $A$ is diagonalizable.

(i) True | False If $A$ is a $3 \times 3$ matrix with three eigenvalues $\lambda_1 = 1$, $\lambda_2 = 5$ and $\lambda_3 = 15$, then $A$ is invertible.

(j) True | False A $3 \times 3$ matrix with real entries must always have at least one real eigenvalue.

2. Find the eigenvalues of $A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

3. Suppose the the graph of characteristic polynomial of an $3 \times 3$ matrix $A$ is given below.




Circle your answer for each of the following:

(a) Is $A$ invertible? | Yes | No | Not Enough Information |

**Explain your choice:**

(b) Is $A$ diagonalizable? | Yes | No | Not Enough Information |

**Explain your choice:**

4. Suppose the characteristic polynomial of a square matrix $A$ is:

$$p(\lambda) = \lambda(\lambda - 1)^2(\lambda + 2)^3$$

(a) Fill in the following table:

| **Eigenvalues of $A$** | **Algebraic Multiplicity** | **Possible Geometric Multiplicities** |
|---|---|---|
| | | |

(b) Is $A$ invertible? Circle your choice: | Yes | No | Not Enough Information |

(c) Is $A$ diagonalizable? Circle your choice: | Yes | No | Not Enough Information |

(d) $\text{tr}(A) = \boxed{\phantom{xxxx}}$

(e) $\det(A) = \boxed{\phantom{xxxx}}$

(f) What is the size of the matrix $A$? $\boxed{\phantom{xxxx}}$

5. Let $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(a) Find the eigenvalues of $A$.

(b) For each eigenvalue from part (a), find a basis for the corresponding eigenspace.

(c) Show that $A$ is diagonalizable by stating a diagonalizing matrix $P$ and diagonal matrix $D$ so that $A = PDP^{-1}$. There is no need to check the last equality, just state what $P$ and $D$ are.

$P =$

$D =$

6. Let $T : \mathbb{R}^3 \to P_2$ defined by

$$T\left(\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}\right) = (a_0 - a_1) + (a_1 - a_2)x + (a_2 - a_0)x^2.$$

(a) Find $[T]_{\mathcal{B}',\mathcal{B}}$ if $\mathcal{B}$ and $\mathcal{B}'$ are the standard bases (i.e., $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{B}' = \{1, x, x^2\}$)

(b) Is $T$ one-to-one (an injection)? | Yes | No |

**Proof:**

(c) Is $T$ an isomorphism? | Yes | No |

**Proof:**

7. Prove that if $A$ and $B$ are similar matrices, then $A^3$ and $B^3$ are also similar matrices.