Math 425 - Graded Problems - Cosets, etc. Name: ______________________________

1. Suppose $G = \{e, a, b, c\}$ is the group defined by the following Cayley table.

$\cdot$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

   Let $H = \langle a \rangle$.

   (a) Find the left cosets of H in G.

   (b) Find the right cosets of H in G.

   (c) Determine $|G{:}H|$

2. Let A, B and C be groups. Suppose A is a proper sugroup of B and B is a proper subgroup of C. If $|A| = 30$ and $|C| = 900$, what are possible orders of B? Explain your answer!

3. Let G be a group with $|G| = 95$. Let $x, y \in G$, where both x and y are nonidentity elements and $|x| \neq |y|$. Prove that the only subgroup of G that contains both x and y is G.

4. Suppose G is a group and $|G| = 8$. Show that G must have an element of order 2.

5. Suppose G is a group and $|G| = 25$. Prove that G is cyclic **or** $g^5 = e$ for all g in G.