Math 425 - Isomorphisms - Part 1 Name: ______________________________

1. Define $\varphi : (\mathbb{R}^+, \cdot) \to (\mathbb{R}^+, \cdot)$ by
$$\forall x \in \mathbb{R}^+, \varphi(x) = \sqrt{x}.$$
Prove that φ is an automorphism.

2. Define $\varphi : (\mathbb{R}^+, \cdot) \to (\mathbb{R}, +)$ by
$$\forall x \in \mathbb{R}^+, \varphi(x) = \log_{10}(x).$$
Prove that φ is an isomorphism.

3. Let $(\mathbb{R}^n, +)$ be the group of vectors in $\mathbb{R}^n$ under componentwise (vector) addition. Define $\varphi : (\mathbb{R}^n, +) \to (\mathbb{R}^n, +)$ by
$$\forall x \in \mathbb{R}^n, \varphi\left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}\right) = \begin{bmatrix} -x_1 \\ -x_2 \\ \vdots \\ -x_n \end{bmatrix}.$$
Prove that φ is an automorphism. This particular automorphism is called *inversion*, can you geometrically see what this automorphism is doing to the vectors in $\mathbb{R}^n$? That is, what is the geometric *action* of this automorphism on $\mathbb{R}^n$? Maybe think about the special cases where $n = 2$ or $n = 3$.

4. Let G be a group. Let $\varphi : G \to G$ be defined by
$$\forall g \in G, \varphi(g) = g^{-1}.$$
Prove G is an automorphism iff G is abelian.

5. Is $\mathbb{Z} \cong 2\mathbb{Z}$? In general is $\mathbb{Z} \cong n\mathbb{Z}$?