Math 425 - Isomorphisms - Part 2 Name: ______________________________

1. Let G_1 and G_2 be groups. Prove that the external direct product

$$G_1 \times G_2 = \{(g_1, g_2) \mid g_1 \in G_1 \text{ and } g_2 \in G_2\}$$

is a group under componentwise multiplication:

$$(g_1, g_2)(h_1, h_2) = (g_1 h_1, g_2 h_2).$$

2. Prove or disprove $\mathbb{Z} \times \mathbb{Z}$ is cyclic.

3. Prove that $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ has seven subgroups of order 2.

4. Prove or disprove $\mathbb{Z}_4 \times \mathbb{Z}_6 \cong \mathbb{Z}_{24}$ (your answer should be short).

5. Prove or disprove $\mathbb{Z}_5 \times \mathbb{Z}_7 \cong \mathbb{Z}_{35}$ (your answer should be short).

6. Let G be a group. Let $H = \{(g, g) \mid g \in G\}$. Show that $H \leq G \times G$. This group is sometimes called the diagonal of $G \times G$.

7. Let e_H be the identity of the group H. Let G be a group. Prove that $G \cong G \times \{e_H\}$. Note, this is the same as proving $G \cong G \times \mathbb{Z}_1$.

8. Consider the groups $\mathbb{C}$ and $\mathbb{R}$ under addition. Show that $\mathbb{C} \cong \mathbb{R} \times \mathbb{R}$.