Math 425 - Normal Subgroups/Factor Groups Name: ______________________
Spring 2015

1. Consider the group $G = \mathbb{Z}$ and the subgroup $H = 4\mathbb{Z}$
   (a) Prove that $4\mathbb{Z}$ is a normal subgroup of $\mathbb{Z}$. That is, $4\mathbb{Z} \lhd \mathbb{Z}$.
   (b) Write out a Cayley table for the factor group $\mathbb{Z}/4\mathbb{Z}$.
   (c) Show that $\mathbb{Z}/4\mathbb{Z}$ is cyclic.
   (d) Show that $\mathbb{Z}/4\mathbb{Z} \cong \mathbb{Z}_4$ (Hint: there are two ways to solve this problem, one is to use part (b) and the other is to use part (c)).
2. Consider $\langle 5 \rangle \leq U(12)$
   (a) Prove that $\langle 5 \rangle \lhd U(12)$
   (b) Write out a Cayley table for the factor group $U(12)/\langle 5 \rangle$
   (c) Is $U(12)$ cyclic?
   (d) Is $U(12)/\langle 5 \rangle$ cyclic?
   (e) Prove or disprove: $U(12)/\langle 5 \rangle \cong \mathbb{Z}_2$
3. Consider $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \;\middle|\; a, b, d \in \mathbb{R}, ad \neq 0 \right\}$
   (a) Prove that $H \leq GL_2(\mathbb{R})$.
   (b) Prove that H is not a normal subgroup of $GL_2(\mathbb{R})$.
   (c) Consider $S = \left\{ \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \;\middle|\; x \in \mathbb{R} \right\}$. Show that $S \leq H$.
   (d) Is S abelian?
   (e) Prove that $S \lhd H$.
   (f) ♠ Optional Challenge: Prove that H/S is abelian. This problem is connected to Ch. 10#7. Can you answer that question as well?