Math 425 - Graded Problems - Cosets, etc. Name: \_ Spring 2015

1. Suppose  $G = \{e, a, b, c\}$  is the group defined by the following Cayley table.

•	e	a	b	c
e	e	a	b	c
a	a	e	С	b
b	b	c	e	a
c	c	$egin{array}{c} a \\ e \\ c \\ b \end{array}$	a	e

Let  $H = \langle a \rangle$ .

- (a) Find the left cosets of H in G.
- (b) Find the right cosets of H in G.
- (c) Determine |G:H|
- 2. Let A, B and C be groups. Suppose A is a proper sugroup of B and B is a proper subgroup of C. If |A| = 30 and |C| = 900, what are possible orders of B? Explain your answer!
- 3. Let G be a group with |G| = 95. Let  $x, y \in G$ , where both x and y are nonidentity elements and  $|x| \neq |y|$ . Prove that the only subgroup of G that contains both x and y is G.
- 4. Suppose G is a group and |G| = 8. Show that G must have an element of order 2.
- 5. Suppose G is a group and |G| = 25. Prove that G is cyclic or  $g^5 = e$  for all g in G.