Answer these questions in your Blue Book (this test paper will not *be graded). Make sure to start each question on a new page. No calculators, cell phones, or anything else with a battery. Make sure to read all directions and show all your work. Answers missing work will lose points.*

1. (a) Find the slope of the line $3y - 2x + 100\pi = 0$.

   (b) Write the equation of the line through the point $(1, \sqrt{2})$ and parallel to the line $3y - 2x + 100\pi = 0$.

   (c) Write the equation of the circle centered at $(-3, 4)$ that passes through the point $(0, 0)$.

2. Find the limits for parts (a)-(c) (make sure to show you work).

   (a) $\lim\limits_{x\to 3} \dfrac{x-3}{x^2-5x+6}$.

   (b) $\lim\limits_{t\to 0} \dfrac{\sqrt{t+1}-1}{t}$.

   (c) $\lim\limits_{h\to 0} \dfrac{\frac{2}{x+h}-\frac{2}{x}}{h}$.

   (d) Write down a function $f(x)$ so that $f'(x) =$ the limit from Part (c). What is the domain of $f(x)$?

3. For this question $f(x) = \begin{cases} \frac{x^2-1}{x+1}, & \text{if } \quad x < -1 \\ 2x, & \text{if } -1 < x \leq 1 \\ x^3 - 2x + 1, & \text{if } \quad 1 < x \end{cases}$.

   (a) List all values where $f(x)$ is discontinuous.

   (b) Explain your answer(s) to part (b), showing that you know what it means for a function to be continuous at the point $x = a$.

4. For this question, you may use any (correct) method to find the indicated derivatives.

   (a) For $f(x) = \sqrt{x}(\sqrt{x} + 2x)$, find $f'(x)$ and $f'(4)$. (Simplify your answer for $f'(4)$.)

   (b) For $y = x^2 + \frac{1}{x} + \cos(x)$, find $\frac{dy}{dx}$.

   (c) For $g(t) = 100t^{500}$, find $g'(1)$.

   (d) For $h(u) = \frac{\sin(u)}{u}$, find $h'(u)$.

5. Find the equation for the line tangent to $f(x) = (\frac{\pi}{4} - x)\cos(x)$ at $x = \frac{\pi}{4}$, (You must evaluate/simplify trigonometric expressions for this question).

① (a) $3y - 2x + 100\pi = 0$

$3y = 2x - 100\pi$

$y = \frac{2}{3}x - \frac{100\pi}{3}$

so the slope is $\boxed{\frac{2}{3}}$ (+5)

(b) $\boxed{y - \sqrt{2} = \frac{2}{3}(x-1)}$ (+5)

(c) radius $= \sqrt{(-3-0)^2 + (4-0)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$ (+5)

so an eqn of the circle would be:

$\boxed{(x+3)^2 + (y-4)^2 = 25}$ (+5)

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② (a) $\lim_{x\to 3} \frac{x-3}{x^2-5x+6} = \lim_{x\to 3} \frac{x-3}{(x-3)(x-2)}$

$\stackrel{AET}{=} \lim_{x\to 3} \frac{1}{x-2}$

$= \frac{1}{3-2} = \boxed{1}$ (+5)

(b) SOL 1: use the conjugate trick:

$\lim_{t\to 0} \frac{\sqrt{t+1} - 1}{t} = \lim_{t\to 0} \frac{(\sqrt{t+1} - 1)}{t} \frac{(\sqrt{t+1} + 1)}{(\sqrt{t+1} + 1)}$

$= \lim_{t\to 0} \frac{(\sqrt{t+1})^2 - 1}{t(\sqrt{t+1} + 1)}$

$= \lim_{t\to 0} \frac{t+1-1}{t(\sqrt{t+1} + 1)} =$

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$$= \lim_{t\to 0} \frac{t}{t(\sqrt{t+1}+1)} \overset{\text{AET}}{=} \lim_{t\to 0} \frac{1}{\sqrt{t+1}+1}$$

$$= \frac{1}{\sqrt{\lim_{t\to 0}(t+1)}+1}$$

$$= \frac{1}{\sqrt{1}+1} = \boxed{\frac{1}{2}} \quad +5$$

(This solution is for future reference. Check it out after we learn the "Chain Rule")

SOL 2 : OR notice this limit looks like the derivative of some function at a point. Let $f(t) = \sqrt{t+1} = (t+1)^{1/2}$

Then notice :

$$f'(0) = \lim_{t\to 0} \frac{\sqrt{t+1} - \sqrt{0+1}}{t-0}$$

$$= \lim_{t\to 0} \frac{\sqrt{t+1}-1}{t}$$ ← This is the limit we are trying to find

$$f'(t) = \tfrac{1}{2}(t+1)^{-1/2}$$ (by the Chain Rule)

and $$f'(0) = \tfrac{1}{2}(1)^{-1/2} = \boxed{\frac{1}{2}}$$

(c) SOL 1 :

$$\lim_{h\to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \lim_{h\to 0} \frac{\frac{2x}{x(x+h)} - \frac{2}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2x - 2(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{2x-2x-2h}{h(x)(x+h)}$$

$$= \lim_{h\to 0} \frac{-2h}{h\,x(x+h)}$$

$$= \lim_{h\to 0} \frac{-2}{x(x+h)} = \boxed{\frac{-2}{x^2}} \quad +5$$

SOL 2 : (basically do part (d) and then finish)
Notice this limit is of the form of the definition of the derivative of some function. Let
$f(x) = \frac{2}{x} = 2x^{-1}$

Then $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$$

This is the limit we are trying to find. So now use the short cut formulas.

$$= 2(-1)x^{-1-1}$$

$$= -2x^{-2}$$

$$= \boxed{\frac{-2}{x^2}}$$

(d) $\boxed{f(x) = \frac{2}{x}}$ +3

The domain of f is: $\boxed{\{x \in \mathbb{R} \mid x \neq 0\}}$ +2

OR

$\boxed{(-\infty, 0) \cup (0, \infty)}$

OR

$\boxed{\mathbb{R} \setminus \{0\}}$

OR

all real numbers except $x=0$

all of these are just different notations for the same thing.

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3 (a) $\boxed{x = -1 \quad \text{and} \quad x = 1}$ +10

(b) f is discontinuous at $x = -1$ because $f(-1)$ does not exist! ($x=-1$ is not part of the domain of the function) +5

Remember : A function f is continuous at $x = a$ if
(1) $f(a)$ is defined (that is, a is in the domain of f)
(2) $\lim_{x \to a} f(x)$ exists
(3) $\lim_{x \to a} f(x) = f(a)$

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So at $x=-1$ the function did not satisfy (1).

f is discontinuous at $x=1$: why?

notice $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 2x = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^3-2x+1) = 0$

$2 \neq 0$ so this tells us $\lim_{x\to 1} f(x)$ does NOT exist

therefore (2) is not satisfied and hence f is discontinuous at $x=1$. (+5)

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(4) (a) $f(x) = \sqrt{x}(\sqrt{x}+2x)$

$= x^{1/2}(x^{1/2}+2x)$

$= x^{1/2+1/2} + 2x^{1+1/2}$

$= x + 2x^{3/2}$

$\boxed{f'(x) = 1 + 2\left(\tfrac{3}{2}\right)x^{1/2} = 1+3x^{1/2} = 1+3\sqrt{x}}$ (+4)

$f'(4) = 1+3\sqrt{4} = 1+3\cdot 2 = \boxed{7}$ (+1)

(b) $y = x^2 + x^{-1} + \cos(x)$

$\frac{dy}{dx} = 2x + (-1)x^{-2} - \sin(x)$

so $\boxed{\frac{dy}{dx} = 2x - x^{-2} - \sin(x)}$ (+5)

(c) $g'(t) = 100 \cdot 500\, t^{499}$

$\boxed{g'(1) = 50000}$ (+5)

(d) $\boxed{h'(u) = \frac{u\cos(u) - \sin(u)}{u^2}}$ (+5)

5

$$f\left(\frac{\pi}{4}\right) = 0 \qquad +5$$

$$f'(x) = \left(\frac{\pi}{4} - x\right)(-\sin(x)) + \cos(x)(-1) \qquad +5$$

$$f'\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)(-1) = \frac{-\sqrt{2}}{2} \qquad +5$$

$$\boxed{y - 0 = \frac{-\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)} \qquad +5$$

OR

$$y = \frac{-\sqrt{2}}{2}x + \frac{\pi\sqrt{2}}{8}$$