Answer these questions in your Blue Book (this test paper will not *be graded). Make sure to start each question on a new page. No calculators, cell phones, or anything else with a battery. Make sure to read all directions and show all your work. Answers missing work will lose points.*

1. (20 pts) Solve the following:
   (a) Find $\frac{dy}{dx}$ if $y = \dfrac{1}{1+\tan(x^2)}$.
   (b) Find $g'(t)$ if $g(t) = \sqrt{t^2(1-t)}$.

2. (16 pts) Find an equation for the line tangent to $y\cos(x - \frac{\pi}{3}) = \pi$ at the point $(\frac{\pi}{3}, \pi)$, (You must evaluate/simplify trigonometric expressions for this question).

3. (20 pts) Find the absolute maximum and minimum values of $f(x) = (x^2 - 1)^3$ on the interval $[-2, 0]$.

4. (16 pts) Suppose $f(x) = \dfrac{x}{x^2+4}$. *You must show work for this question.*
   For parts (a)-(d) if there are none, make sure to write "none" or "nowhere".
   (a) On what interval(s) is f increasing.
   (b) On what interval(s) is f decreasing.
   (c) Write down the x-coordinate of any local maxima.
   (d) Write down the x-coordinate of any local minima.

5. (8 pts) Find the horizontal asymptote(s) of $f(x) = \dfrac{1-3x^3}{4x^3+2x^2-1}$.
   You must show work for this question.

6. (20 pts) A spider is building her web. She has anchored her web to a point on the wall 40 cm below the ceiling, and is crawling across the ceiling (in a straight line, away from the wall) at a constant rate of 1 cm/sec. How fast is the angle her web is making with the ceiling changing when she is 30 cm from the wall? (assume her web is a straight line).

(1) (a)

$$\frac{dy}{dx} = \frac{(1+\tan(x^2))(0) - 1(\sec^2(x^2)2x)}{(1+\tan(x^2))^2}$$

$$\frac{dy}{dx} = \frac{-\sec^2(x^2)2x}{(1+\tan(x^2))^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x\sec^2(x^2)}{(1+\tan(x^2))^2}}$$

(b)

$$g(t) = \sqrt{t^2(1-t)} = \sqrt{t^2-t^3} = (t^2-t^3)^{1/2}$$

$$g'(t) = \frac{1}{2}(t^2-t^3)^{-1/2}(2t-3t^2)$$

$$\boxed{g'(t) = \frac{2t-3t^2}{2\sqrt{t^2-t^3}}}$$

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(2)

We need to find the slope of the tangent line:

Solution 1: differentiate both sides of the implicit equation

$$\frac{d}{dx}\left(y\cos\left(x-\frac{\pi}{3}\right)\right) = \frac{d}{dx}(\pi)$$

$$y\left(-\sin\left(x-\frac{\pi}{3}\right)(1)\right) + \cos\left(x-\frac{\pi}{3}\right)\frac{dy}{dx} = 0$$

$$-y\sin\left(x-\frac{\pi}{3}\right) + \cos\left(x-\frac{\pi}{3}\right)\frac{dy}{dx} = 0$$

now solve for $\frac{dy}{dx}$:

$$\cos\left(x-\frac{\pi}{3}\right)\frac{dy}{dx} = y\sin\left(x-\frac{\pi}{3}\right)$$

$$\frac{dy}{dx} = \frac{y\sin\left(x-\frac{\pi}{3}\right)}{\cos\left(x-\frac{\pi}{3}\right)}$$

evaluate at $(\pi/3, \pi)$:

$$\left.\frac{dy}{dx}\right|_{(x,y)=(\pi/3,\pi)} = \frac{\pi\sin\left(\frac{\pi}{3}-\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{3}-\frac{\pi}{3}\right)} = \frac{\pi\sin(0)}{\cos(0)} = \frac{\pi\cdot 0}{1} = 0$$

equation of tangent line: $\boxed{y-\pi = 0\left(x-\frac{\pi}{3}\right)}$ or $\boxed{y=\pi}$

Solution 2 : first write $y = \dfrac{\pi}{\cos\left(x - \frac{\pi}{3}\right)}$

now differentiate .... etc.

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(3) use the "closed interval method"

first we find the critical numbers of f in the interval $[-2,0]$:

$$f(x) = (x^2-1)^3$$

$$\begin{aligned} f'(x) &= 3(x^2-1)^2 \frac{d}{dx}(x^2-1) \\ &= 3(x^2-1)^2\, 2x \\ &= 6x(x^2-1)^2 \\ &= 6x((x-1)(x+1))^2 \\ &= 6x(x-1)^2(x+1)^2 \end{aligned}$$

$f'(x) = 0 = 6x(x-1)^2(x+1)^2$

$x=0$ OR $x=1$ OR $x=-1$

NOT in the interval $[-2,0]$ some do not consider this to be a critical number.

$f'(x)$ is undefined ... never!

$f'(x)$ is a polynomial.

Now evaluate f at the critical numbers and at the end points of the interval :

$\boxed{f(0) = (0-1)^3 = -1}$ ← absolute min. value

$f(-1) = ((-1)^2-1)^3 = (1-1)^3 = 0$

$\boxed{f(-2) = ((-2)^2-1)^3 = (4-1)^3 = 27}$ ← absolute max. value

4 (a)

$$f'(x) = \frac{(x^2+4)(1) - x(2x)}{(x^2+4)^2} = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{-x^2+4}{(x^2+4)^2} = \frac{-(x^2-4)}{(x^2+4)^2}$$

$$= \frac{-(x-2)(x+2)}{(x^2+4)^2}$$

Find critical numbers of f:

$f'(x) = 0 = -\frac{(x-2)(x+2)}{(x^2+4)^2}$

$0 = -(x-2)(x+2)$

$x = 2$ <u>OR</u> $x = -2$

$f'(x)$ is undefined when $(x^2+4)^2 = 0$ no solutions in $\mathbb{R}$.

So the only critical numbers are $x = 2$ <u>OR</u> $x = -2$

one way to help finish this problem is to do a "sign analysis"

f': − + −

−2 2

intervals: $(-\infty, -2)$ $(-2, 2)$ $(2, \infty)$

so	f is increasing on $(-2, 2)$
(b)	f is decreasing on $(-\infty, -2) \cup (2, \infty)$
(c)	f has a local maxima when $x = 2$
(d)	f has a local minima when $x = -2$

remember the question only asks for the x-coordinates.

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$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1-3x^3}{4x^3+2x^2-1}$$

$$= \lim_{x\to\infty} \frac{(1-3x^3)\left(\frac{1}{x^3}\right)}{(4x^3+2x^2-1)\left(\frac{1}{x^3}\right)}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x^3} - \frac{3x^3}{x^3}}{\frac{4x^3}{x^3} + \frac{2x^2}{x^3} - \frac{1}{x^3}}$$

$$= \frac{\lim_{x\to\infty} \frac{1}{x^3} - \lim_{x\to\infty} 3}{\lim_{x\to\infty} 4 + \lim_{x\to\infty} \frac{2}{x} - \lim_{x\to\infty} \frac{1}{x^3}}$$

$$= \frac{0-3}{4+0-0} = \frac{-3}{4}$$

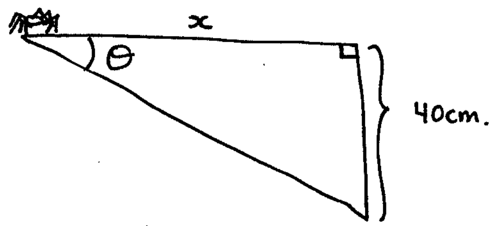
$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1-3x^3}{4x^3+2x^2-1}$$

$$= \frac{\lim_{x\to-\infty} \frac{1}{x^3} - \lim_{x\to-\infty} 3}{\lim_{x\to-\infty} 4 + \lim_{x\to-\infty} \frac{2}{x} - \lim_{x\to-\infty} \frac{1}{x^3}}$$

$$= \frac{0-3}{4+0-0} = \frac{-3}{4}$$

So the only horizontal asymptote of f is $\boxed{y = -\frac{3}{4}}$

(6)



given : $\dfrac{dx}{dt} = 1 \text{ cm/sec.}$

unknown : $\dfrac{d\theta}{dt}$ when $x = 30\text{cm}$.

equation relating x and θ : (there are many possible equations)

$$\cot\theta = \frac{x}{40}$$

(you could have use other trig. relations instead such as $\tan\theta = \dfrac{40}{x}$)

differentiation :

$$\frac{d}{dt}\cot\theta = \frac{d}{dt}\left(\frac{x}{40}\right)$$

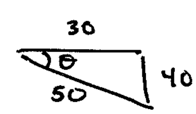
$$-\csc^2\theta\,\frac{d\theta}{dt} = \frac{1}{40}\,\frac{dx}{dt}$$

solve for the unknown :

$$\frac{d\theta}{dt} = \frac{-1}{40\csc^2\theta}\,\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-\sin^2\theta}{40}\,\frac{dx}{dt}$$

substitution : when $x = 30\text{cm}$ the picture looks like :



$$\sin\theta = \frac{40}{50} = \frac{4}{5}$$

so $$\frac{d\theta}{dt} = -\frac{\left(\frac{4}{5}\right)^2}{40}(1) = \frac{-16}{40\cdot 25}$$

$$\boxed{\frac{d\theta}{dt} = \frac{-2}{5\cdot 25} = \frac{-2}{125}\ \text{rad/sec}}$$