

PRACTICE MIDTERM

Math 330 - Number Systems (Introduction to Higher Mathematics)

March 22, 2012

Name: _____

When you are finished please sign the following:

Signature: _____

By signing my name I pledge that I have not broken the Student Academic Honesty Code at any point during this examination.

Read all of the following information before starting the exam:

- You have 1 hour and 10 minutes to complete the exam.
- This is an **open-book** exam, i.e., you may use your class notes and your textbook during this test.
- Show all work, clearly and in order, if you want to get full credit. Please make sure you read the directions for each problem. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- The examination is scored out of 60 points and has **five** problems. There is also a bonus problem worth 5 points. Your usual score will be the sum of the scores for the four best answers and the bonus answer. Your total score will be minimum of 60 and your usual score. Thus, the maximum possible score is 60 points.
- Good luck!

5 1. (a) Using only results before Proposition 1.22, prove that for all $m \in \mathbb{Z}$, $-(-m) = m$.

10 (b) Let $m \in \mathbb{N}$ and $n \in \mathbb{Z}$. If $mn \in \mathbb{N}$, then $n \in \mathbb{N}$.

15 2. Let $(s_k)_{k=0}^{\infty}$ be the sequence defined by

$$\begin{aligned}s_0 &= 0, \\ s_1 &= 1, \text{ and} \\ \forall k \geq 2, s_k &= 3s_{k-1} - 2s_{k-2}.\end{aligned}$$

Show: $\forall n \in \mathbb{Z}_{\geq 0}, s_n = 2^n - 1$.

15 3. Prove or give a counterexample:

Let X be a set. If $A, B, C \subseteq X$, then

$$A - (B \cup C) = (A - B) \cap (A - C).$$

15 4. Let $k \in \mathbb{N}$. Show:

$$\sum_{j=1}^k j^2 = \frac{k(k+1)(2k+1)}{6}.$$

5. Let \sim be the relation on \mathbb{Z} given by

$$m \sim n \text{ if and only if } m^2 + n^2 \text{ is even.}$$

Prove or disprove the following:

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(a) \sim is reflexive.

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(b) \sim is symmetric.

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(c) \sim is transitive.

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(d) Is \sim an equivalence relation?

6. **BONUS:** You will only see this on the actual midterm!