Math 371 - Ordinary Differential Equations

Nate Reff

August 30, 2011

Definition of a Differential Equation

Definition

A **differential equation (DE)** is an equation containing derivatives of one or more dependent variables with respect to one or more independent variables

Examples:

1. $\dfrac{dy}{dx} = 5y,$
2. $\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = 10,$
3. $y'' - 2yx = 0.$
4. $\dfrac{dx}{dt} + \dfrac{dy}{dt} = xy.$

Types of Differential Equations

Definition

An **ordinary differential equation (ODE)** is an equation that only contains ordinary derivatives of one or more dependent variables with respect to a single independent variable.

Examples: All of the previous examples are ODEs.

Definition

A **partial differential equation (PDE)** is an equation involving the partial derivatives of one or more dependent variables of two or more independent variables.

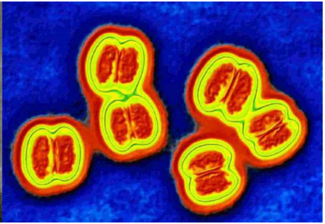
Example: $\dfrac{\partial u}{\partial x} + \dfrac{\partial^2 u}{\partial y^2} = 0.$ (Math 471)

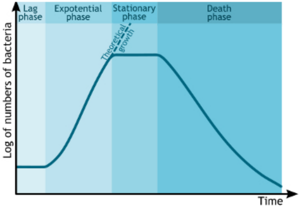
Example 1 - Population Growth

$P(t)$ = **total population** at time t.

Thomas Malthus (1798)

$$\frac{dP}{dt} \propto P \qquad \text{or} \qquad \boxed{\frac{dP}{dt} = kP}.$$



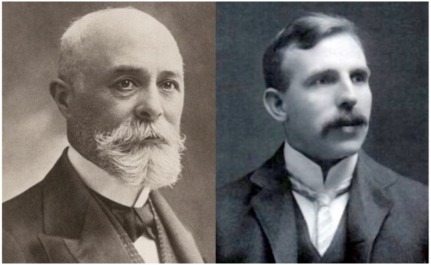


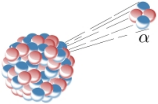
Example 2 - Radioactive Decay

$N(t)$ = **quantity** at time t.

Henri Becquerel, Ernest Rutherford

$$-\frac{dN}{dt} \propto N \qquad \text{or} \qquad \boxed{\frac{dN}{dt} = -kN}.$$



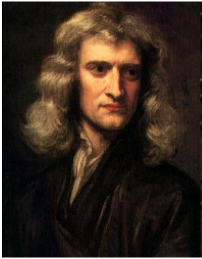


Example 3 - Newton's Law of Cooling

$T(t)$ = **temperature** of object at time t.
T_m = temperature of surrounding medium

Isaac Newton

$$\frac{dT}{dt} \propto T - T_m \qquad \text{or} \qquad \boxed{\frac{dT}{dt} = k(T - T_m)}.$$



Example 4 - Heat Equation

$u(x, y, z, t) =$ **temperature** of object located at (x, y, z) at time t.
$\alpha =$ **thermal diffusivity** (constant for a uniform matertial).

Joseph Fourier

$$\frac{\partial u}{\partial t} - \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 0.$$